Practical Applications of 3D and 4D Filters

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- 3D and 4D Filter Transfer Functions
- Potential Applications of 3D and 4D Filters
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- Seven Practical Demonstrations of 3D/4D Filter Applications

Defining MD Linear Trajectory and MD Plane Wave Signals

$x(t_1, t_2, t_3, \ldots t_M)$ is a MD linear trajectory (LT) signal if there exists a direction $\mathbf{v} \in \mathbb{R}^{\mathbf{M}}$ such that

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_M \end{bmatrix}$$

$$\frac{\partial x(t_1, t_2, t_3, \ldots t_M)}{\partial \mathbf{v}} = 0 \qquad \forall (t_1, t_2, t_3, \ldots t_M) \in \mathbb{R}^M$$

Defining MD Linear Trajectory and MD Plane Wave Signals

$x(t_1, t_2, t_3, \ldots t_M)$ is an MD plane wave (PW) if it can be expressed in the form

$$x(t_1, t_2, t_3, \ldots t_M) = x_{PW}(d_1 t_1 + d_2 t_2 + d_3 t_3 + \ldots d_M t_M)$$

One Reason Why MD Plane Waves Are Important

$$X(j\boldsymbol{\omega}) \equiv \int_{\forall \mathbf{t} \in \mathbb{R}^M} x(\mathbf{t})\, e^{-j\omega_1 t_1}\, e^{-j\omega_2 t_2}\, e^{-j\omega_3 t_3} \ldots e^{-j\omega_M t_M}\, d\mathbf{t}$$

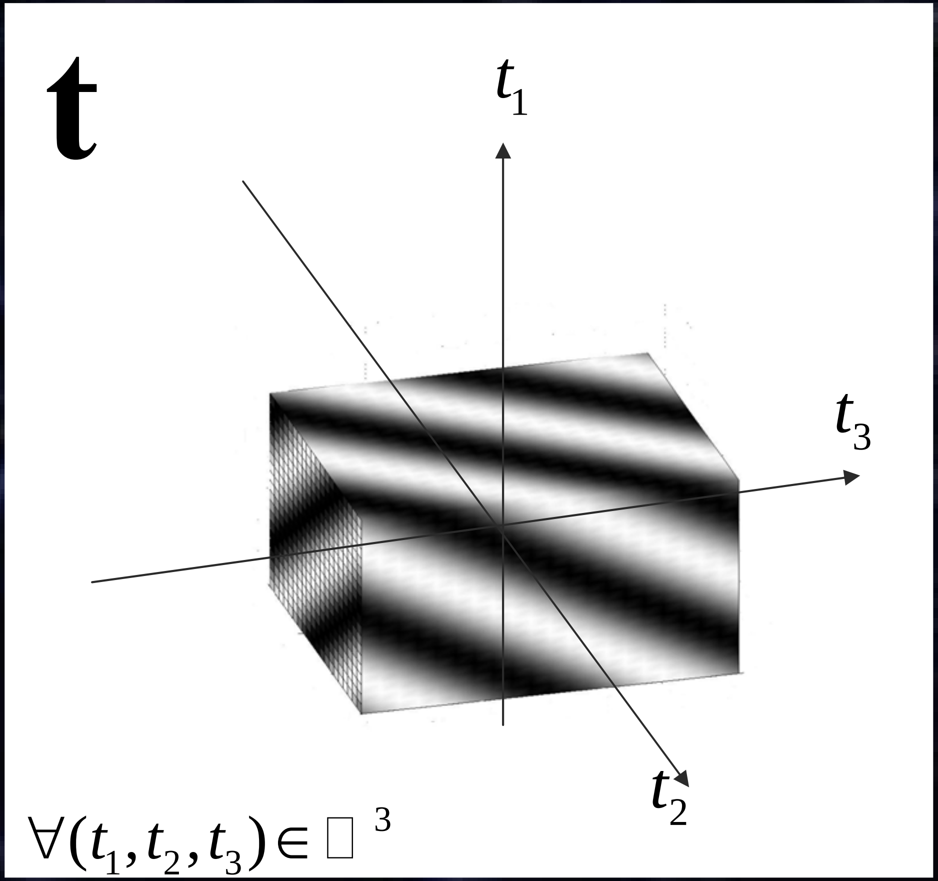
The basis function of the MD Fourier Transform is a MD plane wave:

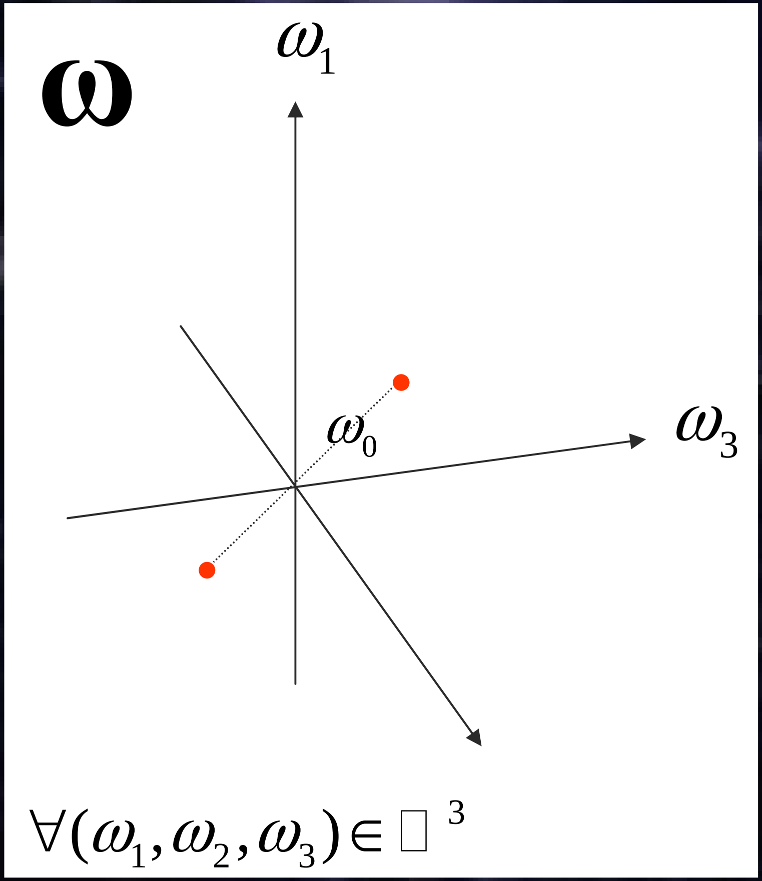
$$e^{-j(\omega_1 t_1 + \omega_2 t_2 + \ldots \omega_M t_M)}$$

Example: The Real part of the 3D basis function

$$\cos(\omega_1 t_1 + \omega_2 t_2 + \omega_3 t_3) \Leftrightarrow \frac{\pi}{2}\delta(\omega - \omega_0) + \frac{\pi}{2}\delta(\omega + \omega_0)$$

$$\omega_0 = \sqrt{\omega_1{}^2 + \omega_2{}^2 + \omega_3{}^2}$$

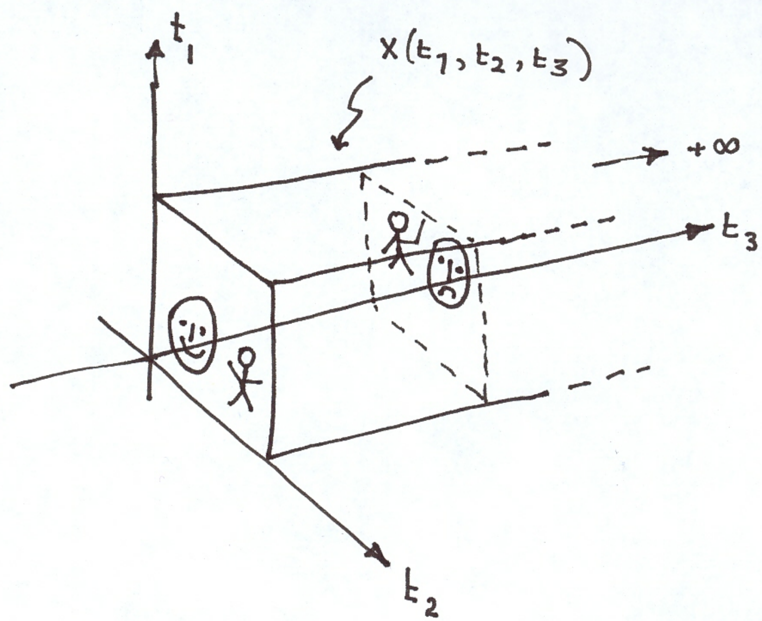




The 3D Spatio-temporal Linear Trajectory

Object Signals

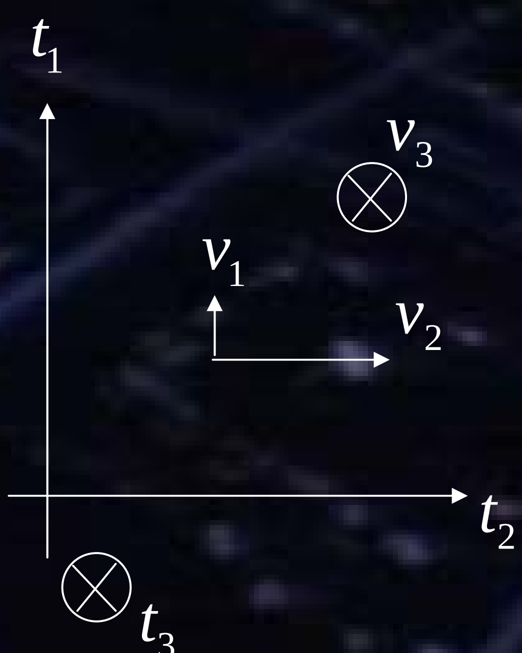
$$\frac{\partial x(t_1,t_2,t_3)}{\partial \mathbf{v}} = 0 \qquad \forall (t_1,t_2,t_3) \in \mathbb{R}^3 \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



Shown only for positive time

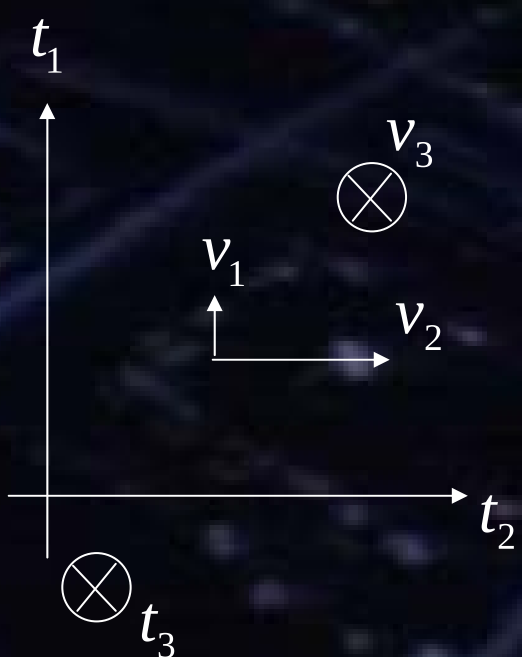
Example: 3D Linear Trajectory Object Signals

$$\frac{\partial x(t_1,t_2,t_3)}{\partial \mathbf{v}} = 0 \qquad \forall (t_1,t_2,t_3) \in \mathbb{R}^3 \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



Another example: 3D Linear Trajectory Object Signals

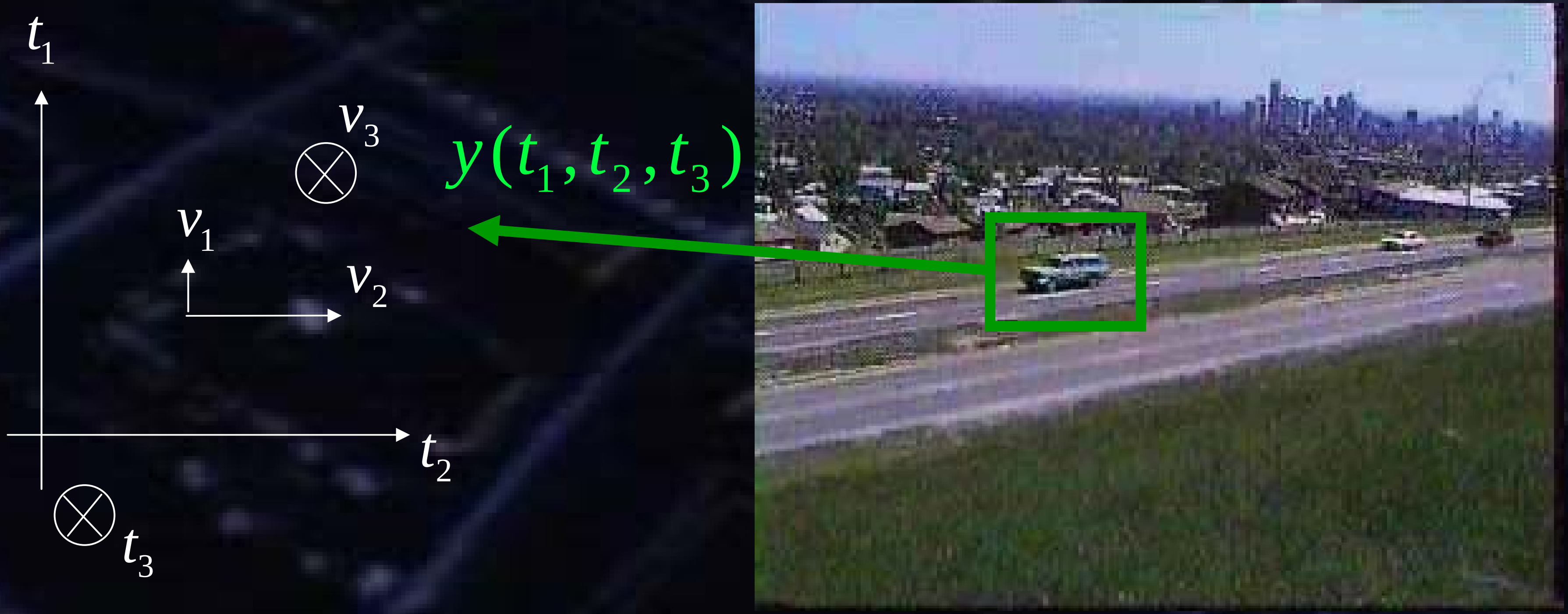
$$\frac{\partial x(t_1, t_2, t_3)}{\partial \mathbf{v}} = 0 \qquad \forall (t_1, t_2, t_3) \in \mathbb{R}^3 \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$



3D Linear Trajectory Signals

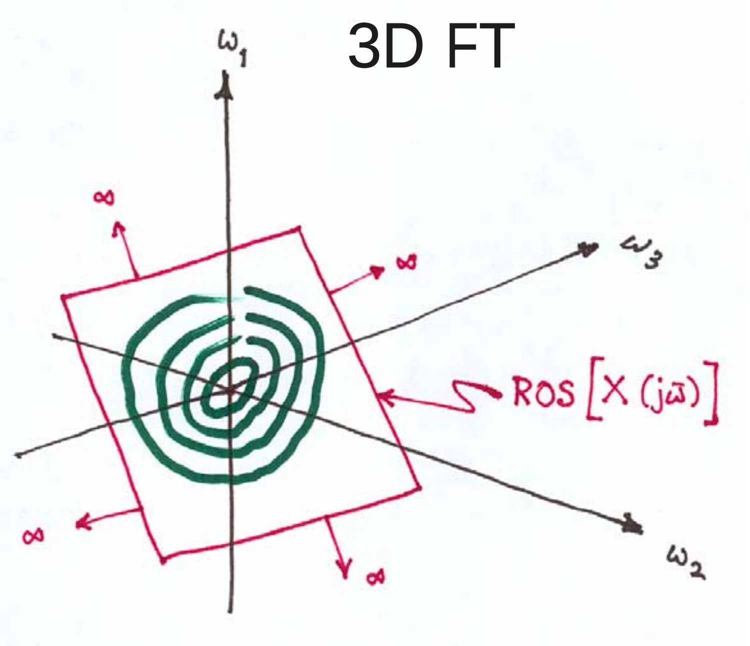
A First Derivative Approximation

SOF: $$v_1 \frac{\partial[y(t_1,t_2,t_3)]}{\partial t_1} + v_2 \frac{\partial[y(t_1,t_2,t_3)]}{\partial t_2} + \frac{\partial[y(t_1,t_2,t_3)]}{\partial t_3} = 0$$



The 3D Fourier Transform of Linear Trajectory Signals

3D FT



3D velocity vector

3D Planar Support

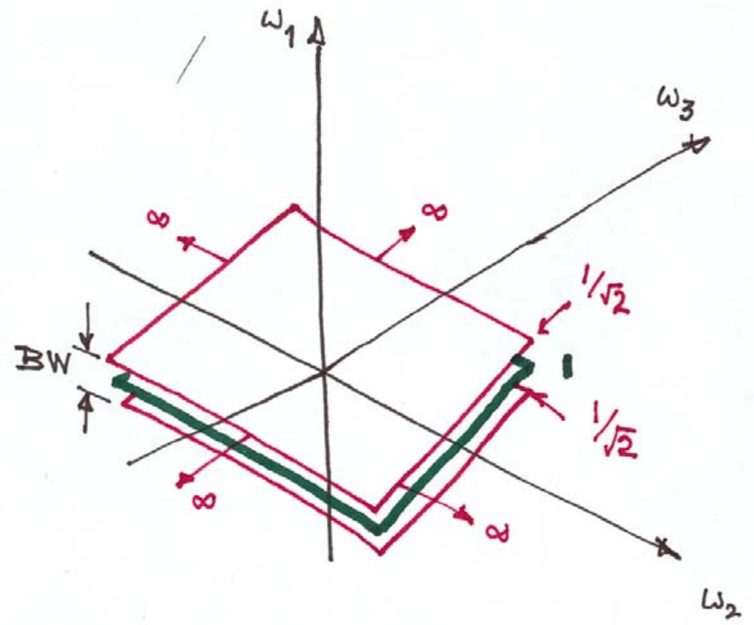
The 3D Fourier Transform of Linear Trajectory Signals

$$\frac{\partial x(t_1, t_2, t_3)}{\partial \mathbf{v}} = 0 \qquad \forall (t_1, t_2, t_3) \in \mathbb{R}^3 \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

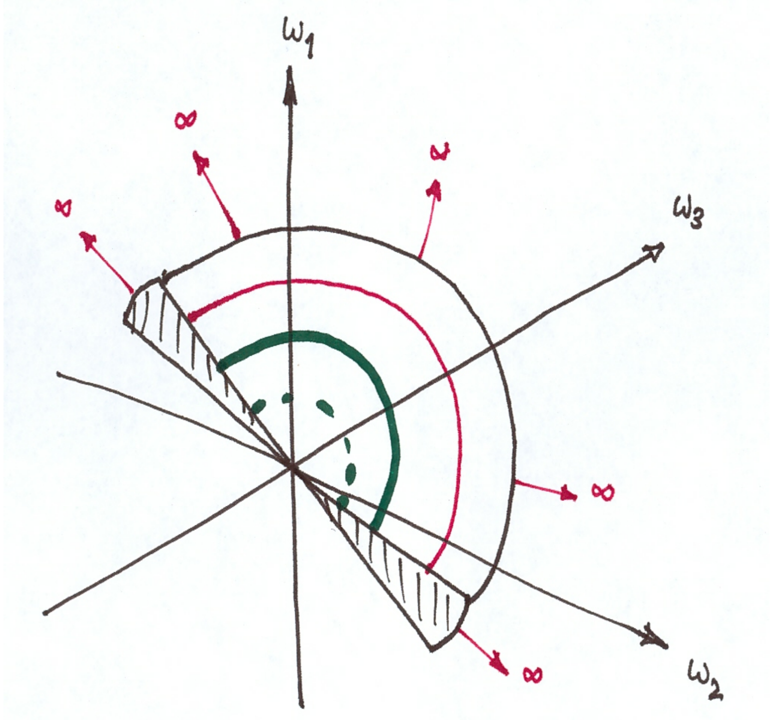
The ROS of the 3D Fourier transform, and therefore the 3D energy density function, of an ideal Linear Trajectory signal lies on a 3D plane in the frequency domain having a normal given by the velocity vector.

We therefore need frequency-planar pass bands to filter ideal Linear Trajectory signals.

Possible 3D Pass Bands for Selectively Filtering Linear Trajectory Signals



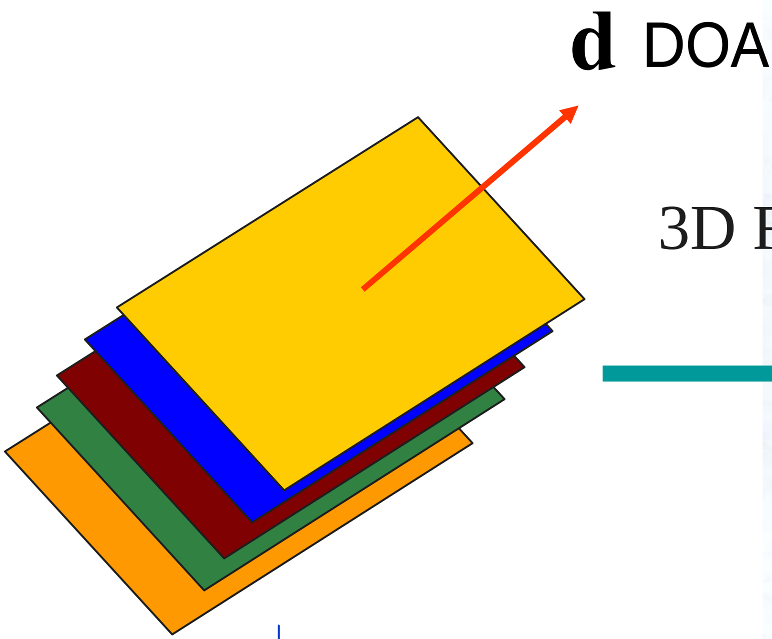
Frequency Planar (FP) - Uniform Bandwidth



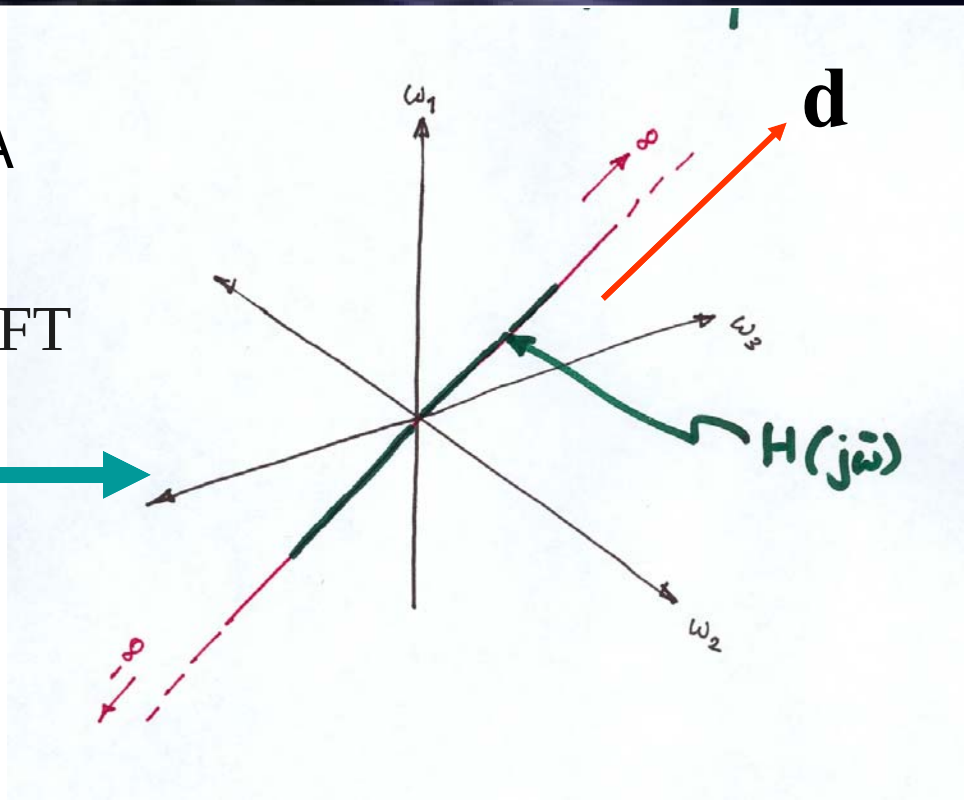
Frequency Planar (FP) – Non-uniform Bandwidth

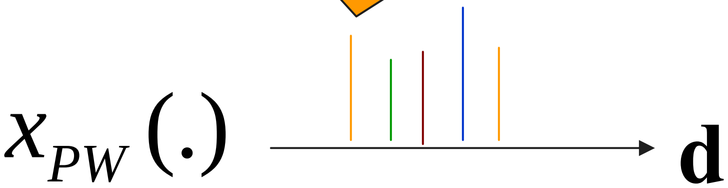
The 3D Fourier Transform of Plane Wave Signals

$$x(t_1, t_2, t_3) = x_{PW}(d_1 t_1 + d_2 t_2 + d_3 t_3)$$



3D FT





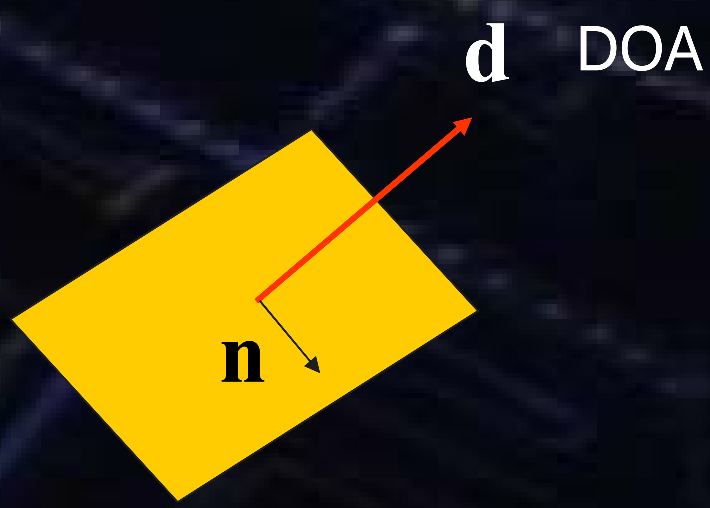
The 3D Fourier Transform of Plane Wave Signals

The ROS of the 3D Fourier transform, and therefore the 3D energy density function, of an ideal 3D plane wave lies on a 3D line through the origin in the frequency domain having a direction given by the DOA.

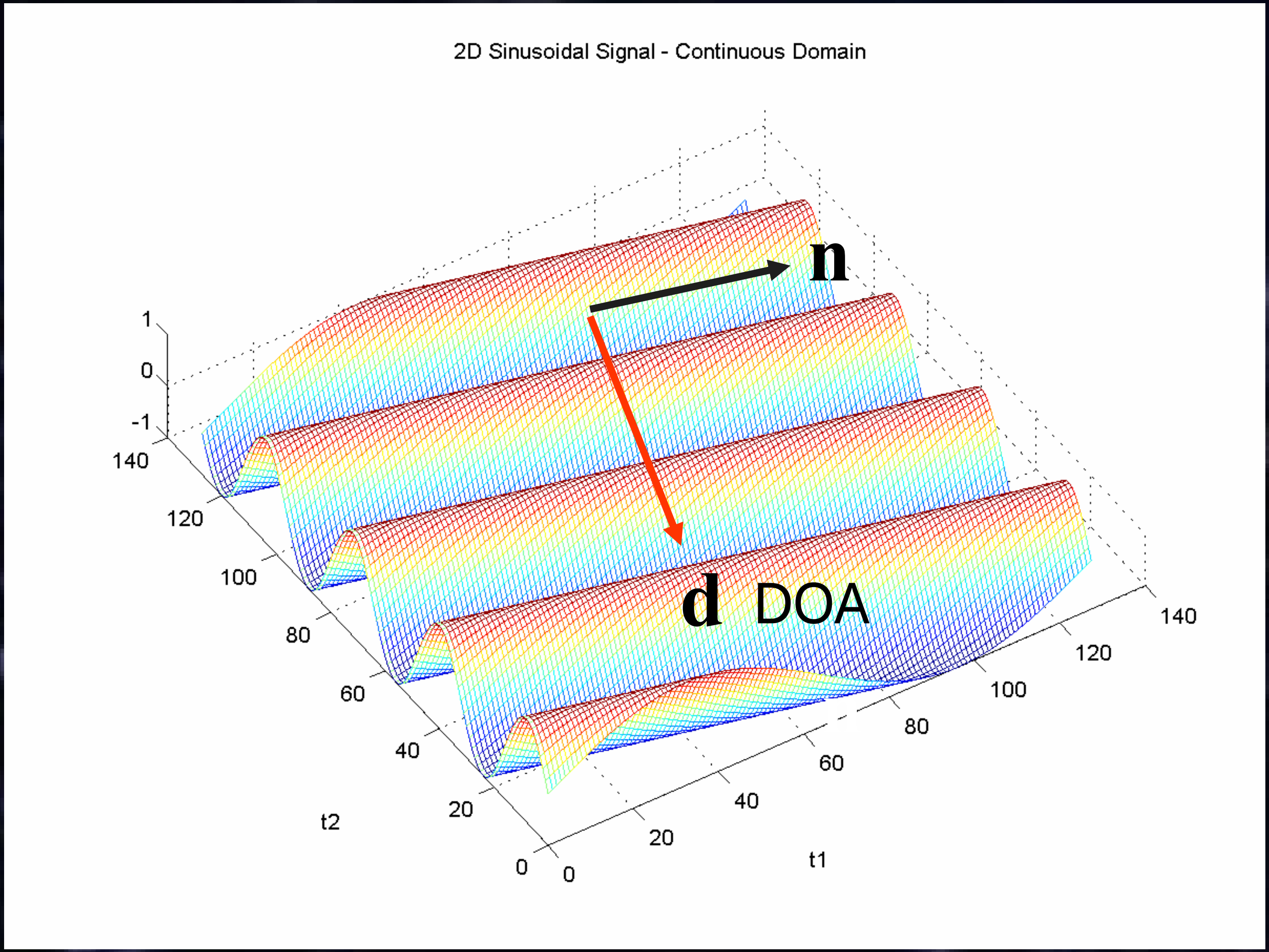
We therefore need frequency-beam/cone shaped pass bands to filter ideal 3D plane wave signals.

On the Relation between 3D Plane Wave Signals and 3D Linear Trajectory Signals

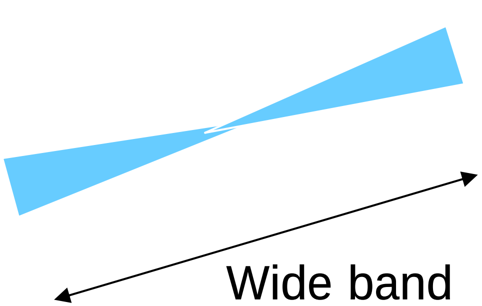
We note that a 3D Plane Wave Signal is a special case of a 3D Linear Trajectory Signal for which every in-plane vector **n** is in the direction of zero-gradient



2D Plane Wave Signals



In the MD case, pass bands are required to have high-selectivity in specific directions and no selectivity in other directions



The Arithmetic Complexity Problem

Order and Image Size

3D FIR Narrow Cone Filters with No Attention to Directionality of the 3D Unit Impulse Response

- to achieve directional 3D Cone Pass Band band widths of a few degrees, the order of a highly-selective 3D FIR Cone Filter is about (128,128,128)

- for image processing applications, there are typically at least 512x512 pixels per frame, implying approximately (128)(128)(128)(512)(512) add/multiply operations per frame

- this level of computational complexity has impeded the applications of highly-selective 3D Cone, Beam and Frequency-Planar filters

Know Your Impulse Response

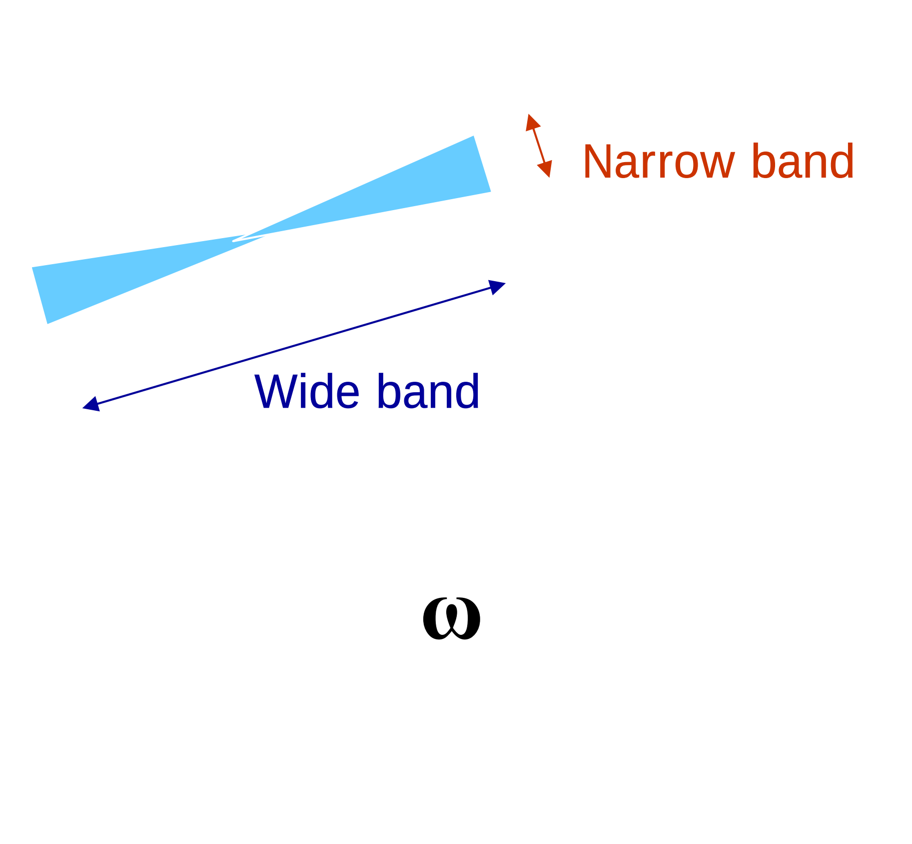
Especially if you must use FIR implementations

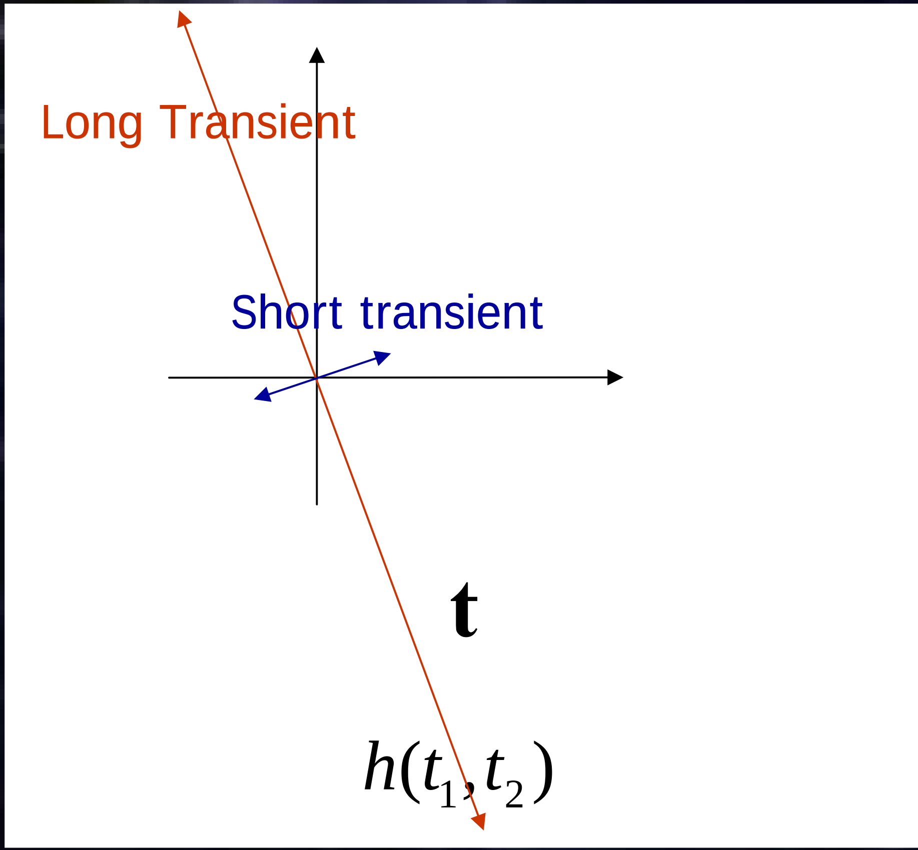
The Unit Impulse Response of 3D Transfer Functions having Highly Directional Pass Bands

The unit impulse response $h(t_1, t_2, t_3)$ has

- ❑ long duration in directions that correspond to narrow directional bandwidths and
- ❑ short duration in directions that correspond to wide directional bandwidths

Take Advantage of the The Long and the Short of it





Knowing The Directionalities of the Transients of the 3D Impulse Response of a 3D Cone Filter can Reduce Computational Complexity by over Two Orders of Magnitude

IIR Implementations in 1D

- In many highly-selective 1D signal processing applications, IIR implementations have been used to reduce complexity and increase processing speeds (e.g. SC LDI ladder and Biquadratic filters, digital WDFs, etc.)

- Pseudo passivity underlies many of the above 1D methods because stability and insensitivity to finite-precisions effects are superior

- Applying similar ideas in MD is viable

For example, 3D FIR order

512^2

Designing 3D IIR Filters

- Reduced Complexity is a primary reason for IIR 3D filters in image processing
- Low-order IIR implementations can achieve narrow 3D pass bands and thereby the long 3D unit impulse response
- PROBLEM: STABILITY

$$Z[h(\mathbf{n})] \equiv H(\mathbf{z}) = \frac{N(a_{ijk}, \mathbf{z})}{D(b_{ijk}, \mathbf{z})}$$

$$H(e^{j\boldsymbol{\omega}}) = H(\mathbf{z})\Big|_{\mathbf{z}=e^{j\boldsymbol{\omega}}}$$

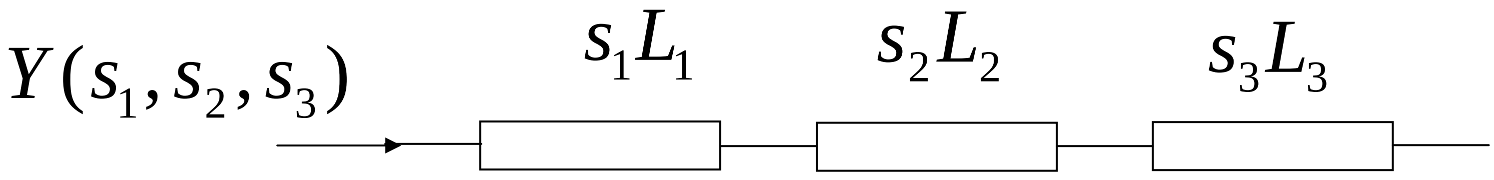
Solution to the Stability Problem: Employ the Characteristic Equations of MD Pseudo-passive Prototype Networks in the Denominator

$$D(\mathbf{z}, y_{ij}) \leftarrow \begin{vmatrix} 1 & y_{12} & y_{13} & y_{14} & y_{15} & y_{16} & y_{17} \\ -y_{12} & s_1 & y_{23} & y_{24} & y_{25} & y_{26} & y_{27} \\ -y_{13} & -y_{23} & s_1 & y_{34} & y_{35} & y_{36} & y_{37} \\ -y_{14} & -y_{24} & -y_{34} & s_2 & y_{45} & y_{46} & y_{47} \\ -y_{15} & -y_{25} & -y_{35} & -y_{45} & s_2 & y_{56} & y_{57} \\ -y_{16} & -y_{26} & -y_{36} & -y_{46} & -y_{47} & s_2 & y_{67} \\ -y_{17} & -y_{27} & -y_{37} & -y_{47} & -y_{57} & -y_{67} & s_2 \end{vmatrix}$$

$$s_{1,2} = (z_{1,2} - 1)/(z_{1,2} + 1)$$

Order: (2,4)

First-order 3D IIR Example: Pseudo-passive 3D Frequency-planar Resonance



The 3D transform impedance is given by the series connection

$$(s_1 L_1 + s_2 L_2 + s_3 L_3)$$

First-order 3D IIR Example: Pseudo-passive 3D Frequency-planar Resonance

$$(s_1L_1 + s_2L_2 + s_3L_3)$$

self resonates in a 3D frequency plane where

$$(\omega_1L_1 + \omega_2L_2 + \omega_3L_3) = 0$$

To Selectively Filter Linear Trajectory Signals

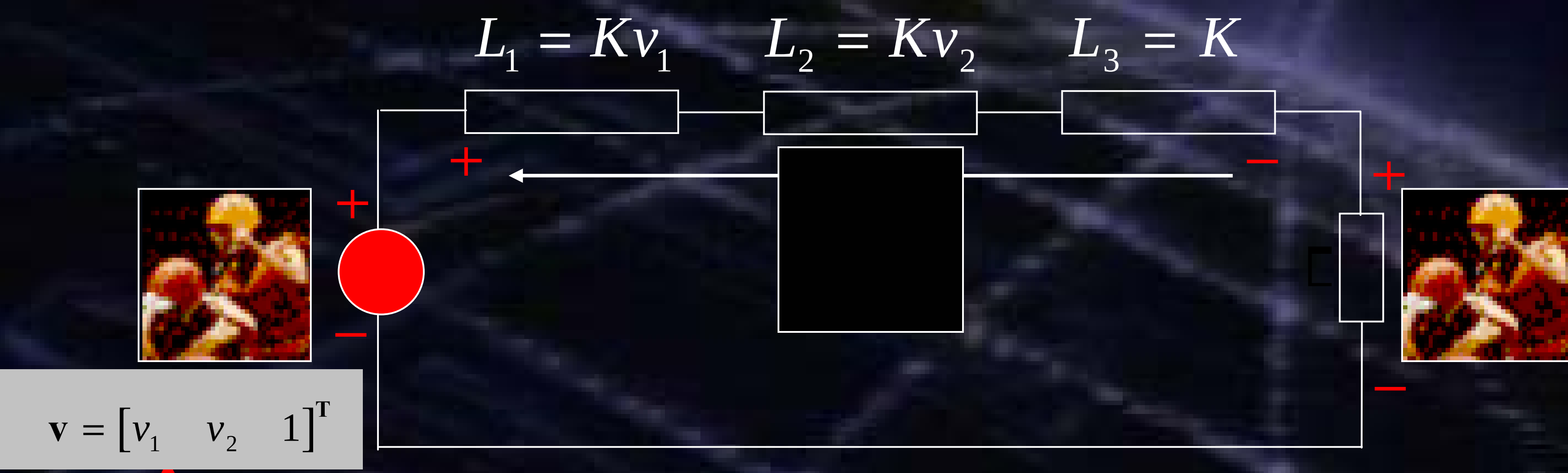
$$L_1 = Kv_1 \qquad L_2 = Kv_2 \qquad L_3 = K$$

ensures that all linear trajectory signals having velocity

$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & 1 \end{bmatrix}^{\mathrm{T}}$$

induce series resonance of the 3D inductance.

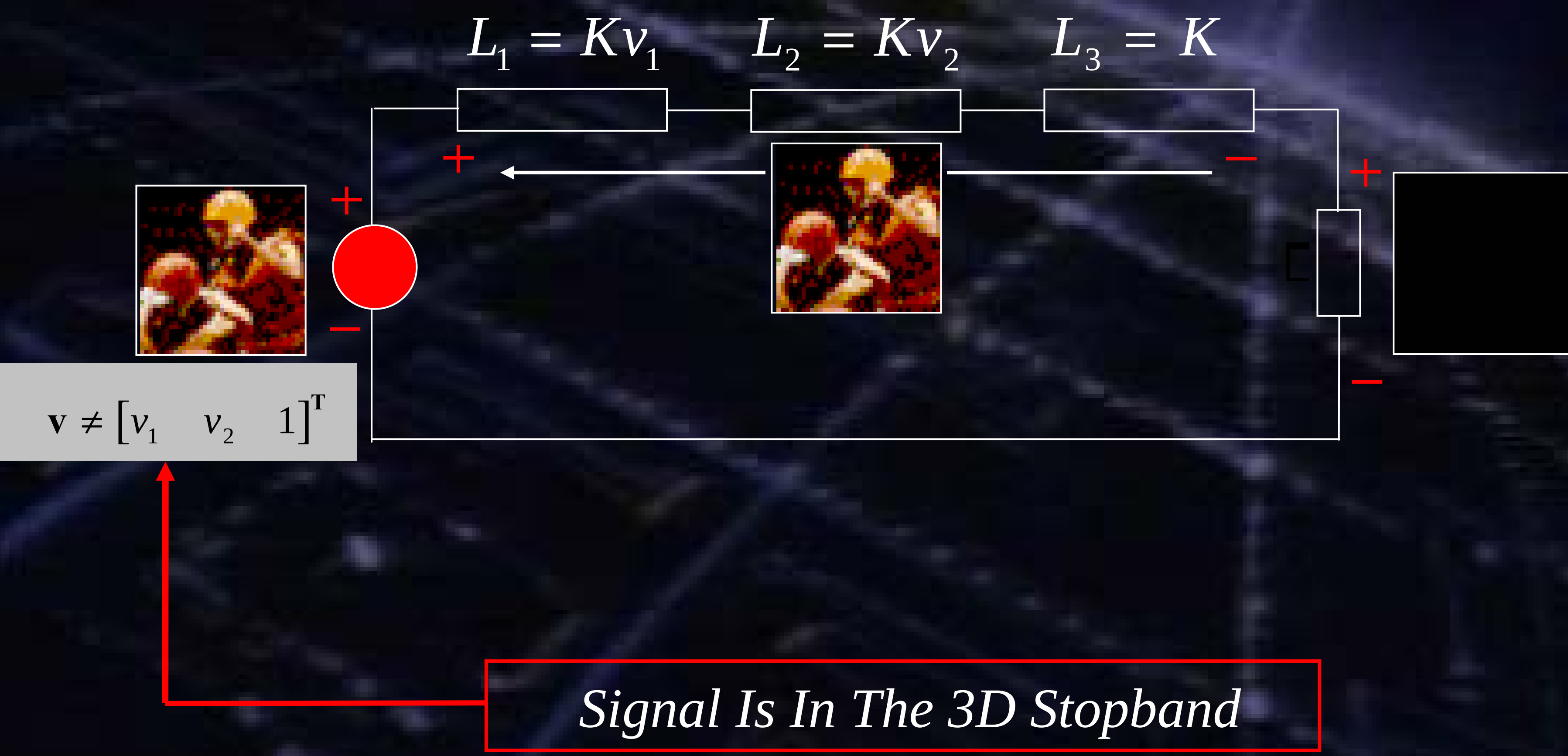
3D Voltages At Resonant Velocity in a Pseudo passive First-order Frequency-Planar (FP) Filter Network



Kirchoff’s Voltage Law at 3D Resonance

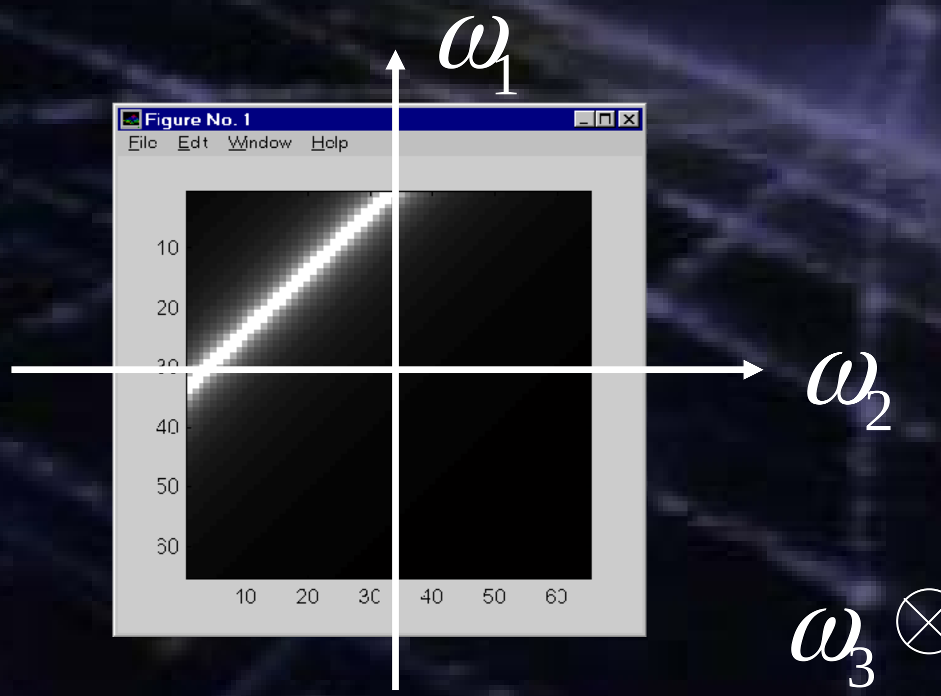
Signal Is In The 3D Passband

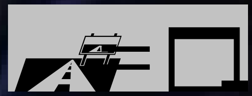
3D Voltages At Non-Resonant Velocity in a Pseudo passive Frequency Planar Filter Network - Idealized

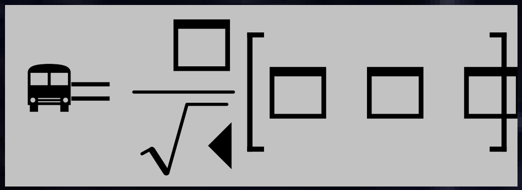


Gain of the Frequency-Planar Filter

$$\left| \frac{1}{1 + jK(\omega_1 v_1 + \omega_2 v_2 + \omega_3)} \right|$$





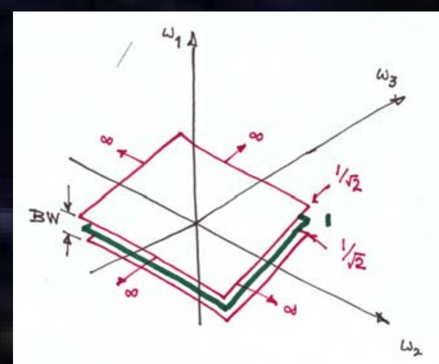


Application 1 - Image Enhancement

input voxels

output voxels

3D FP IIR Filter



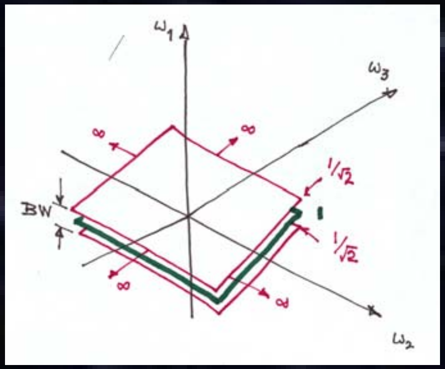
$x(n_1, n_2, n_3)$

$y(n_1, n_2, n_3)$

Application 2 - Image Rejection

input voxels

3D FP IIR Filter



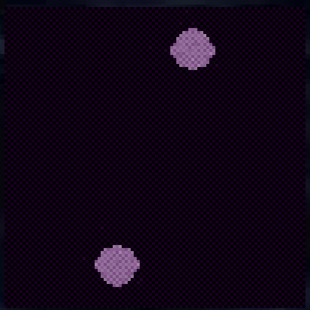
output voxels

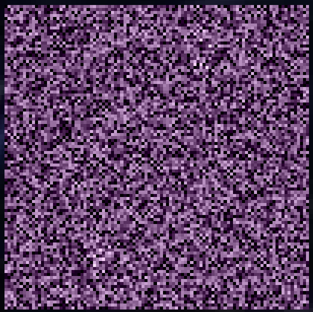
$x(n_1, n_2, n_3)$

$y(n_1, n_2, n_3)$

Application 3 – Object Tracking

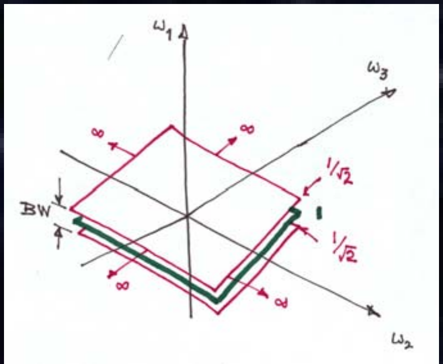
input voxels



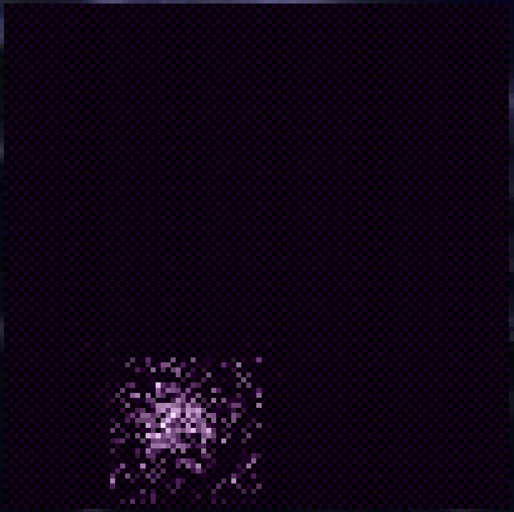


$x(n_1, n_2, n_3)$

3D FP IIR Filter



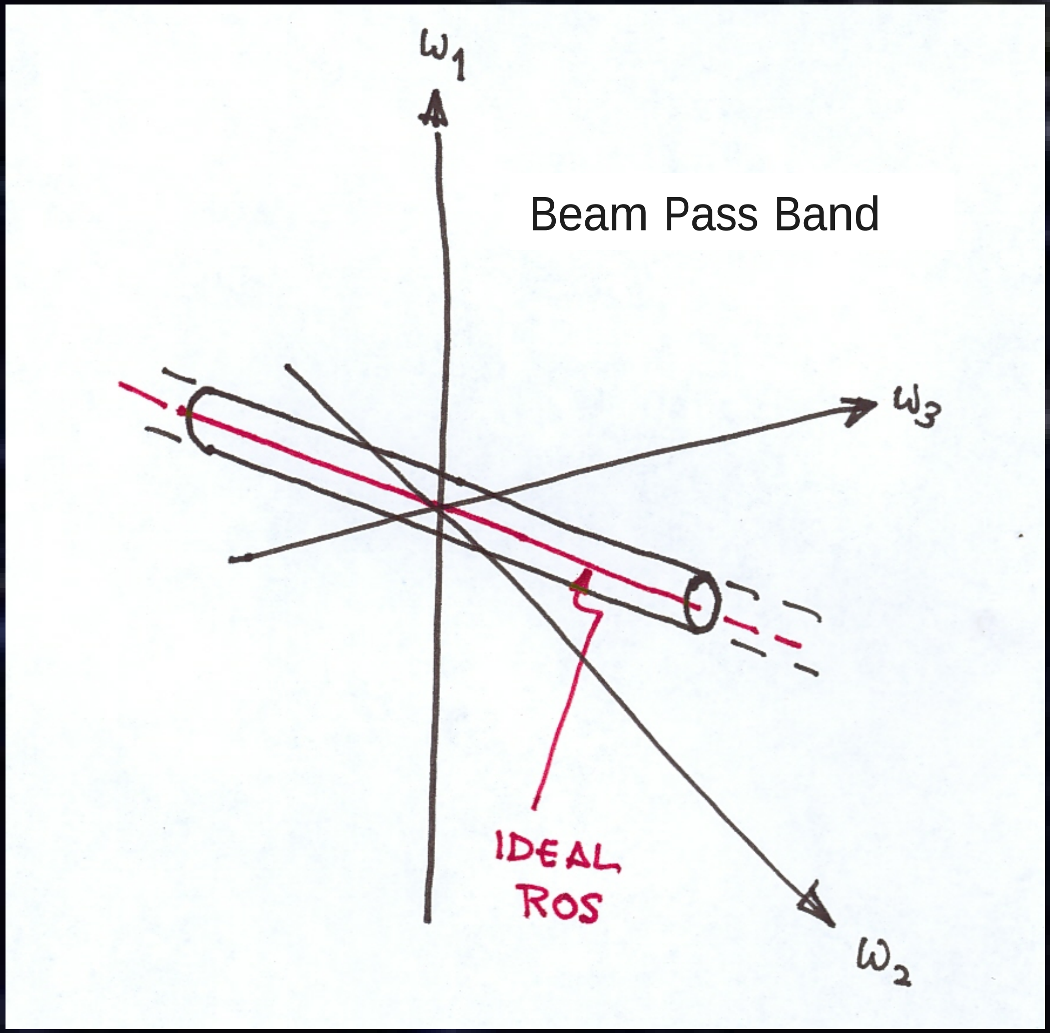
output voxels

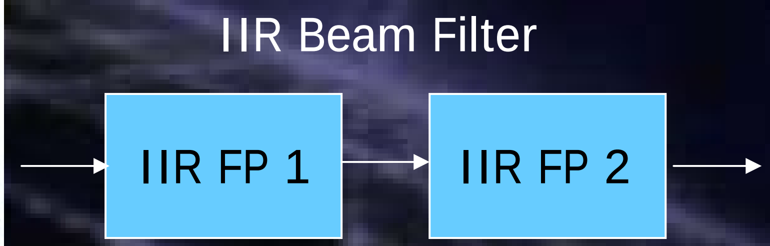


$y(n_1, n_2, n_3)$

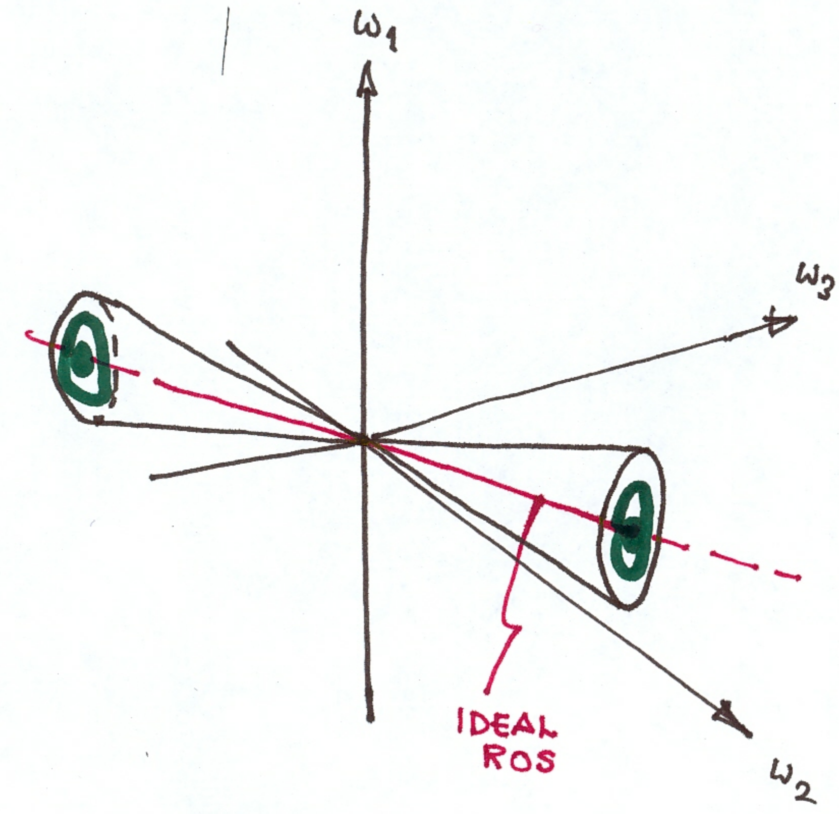
3D Spatio-temporal 3D Beam Filters by Cascading 3D Frequency-planar Filters

3D Spatio-temporal 3D Beam Filters by Cascading 3D Frequency-planar Filters



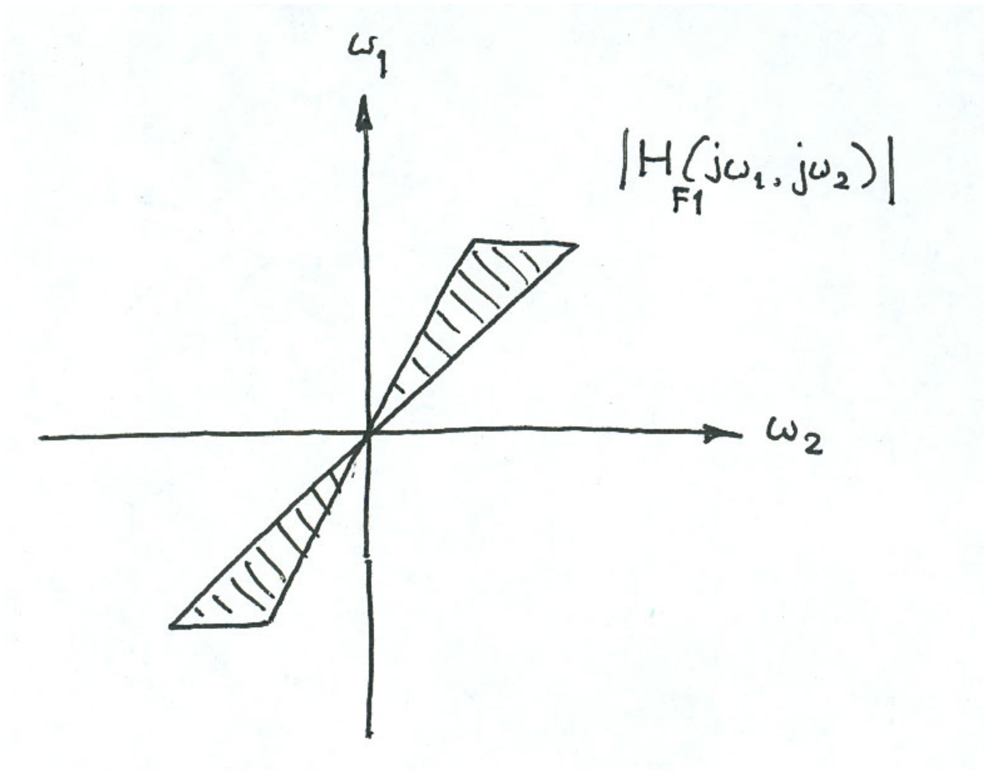


3D Cone Pass Bands are Preferred Pass Bands for many Plane Wave Signals



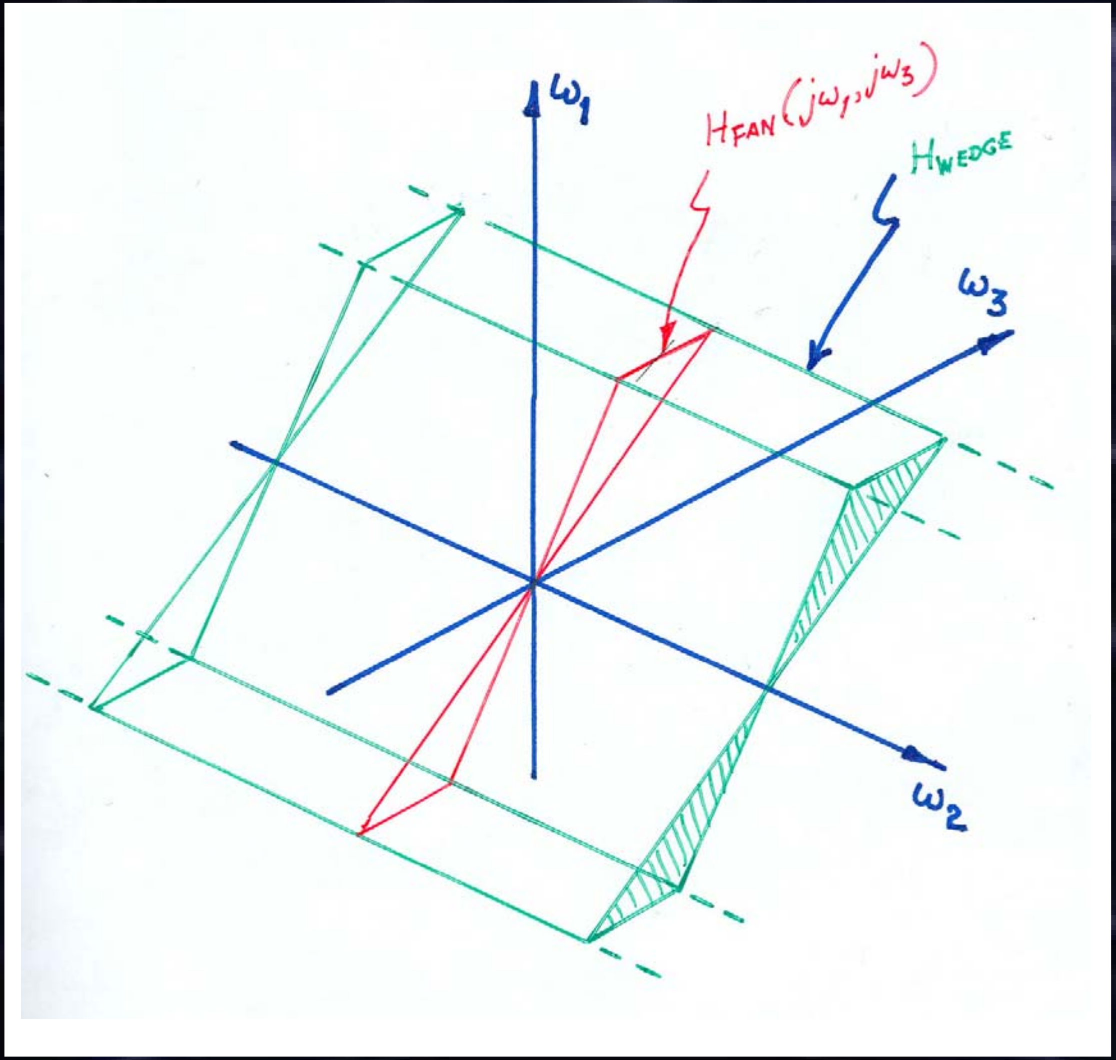
Cone Pass Band

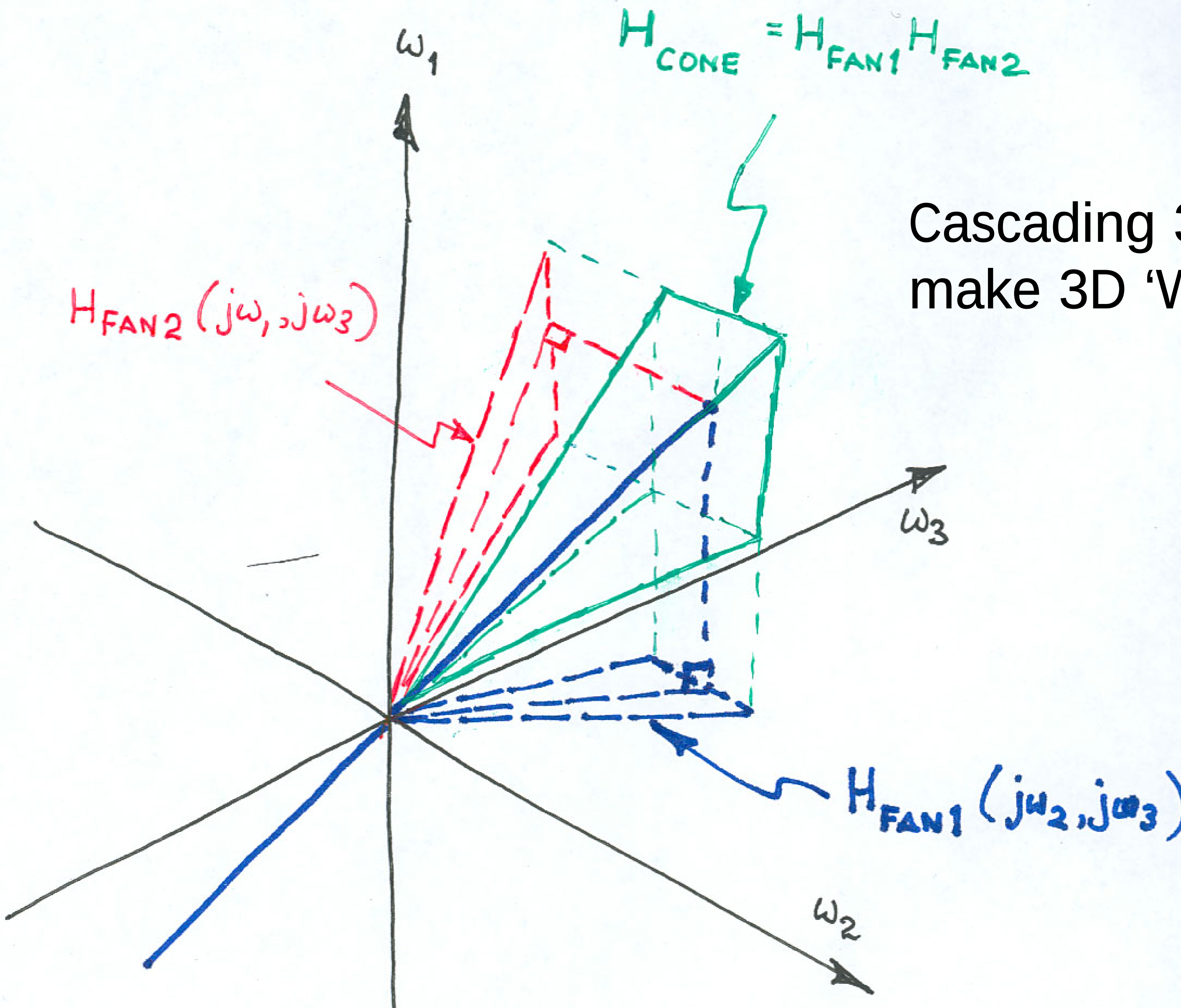
Approximating 3D Cone Pass Bands for Plane Wave Signals Using 3D Cascaded Wedges



2D Fan Pass Band

3D Wedges Using 2D Fans with 3D Signals

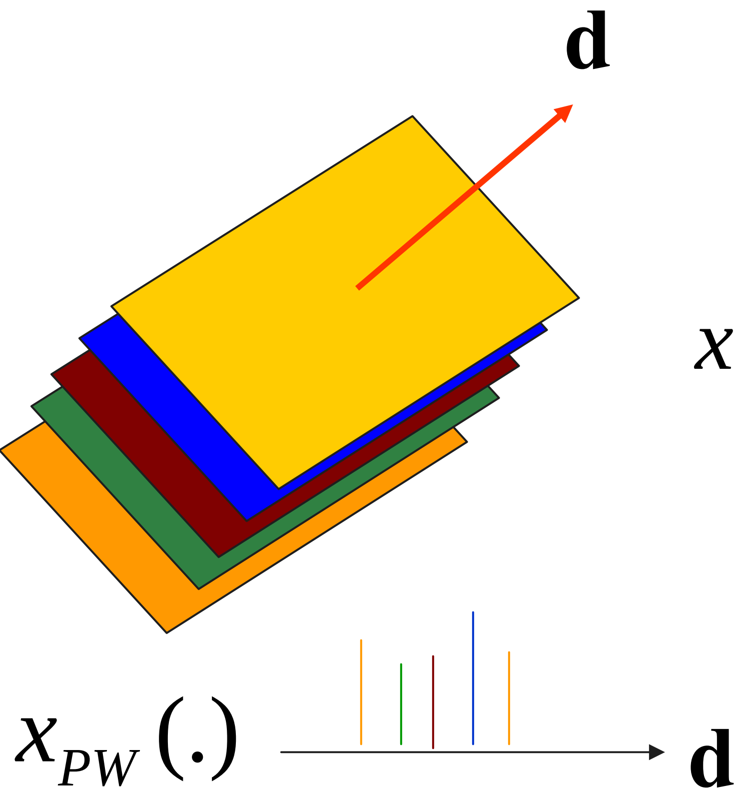


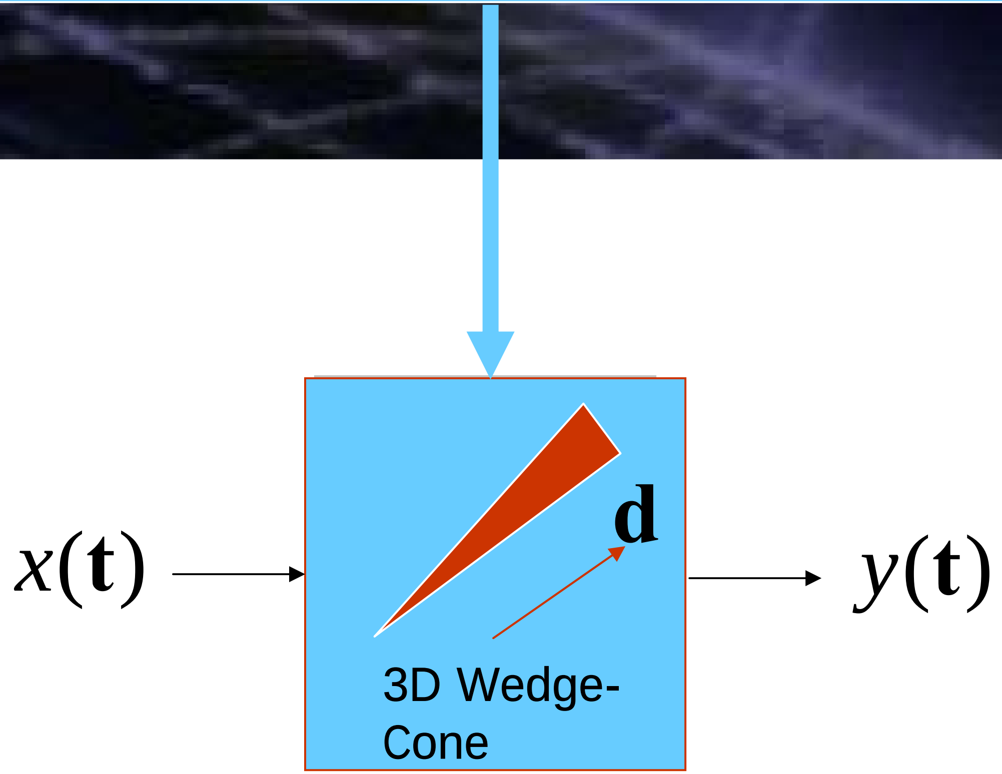


Cascading 3D We
make 3D 'Wedge-

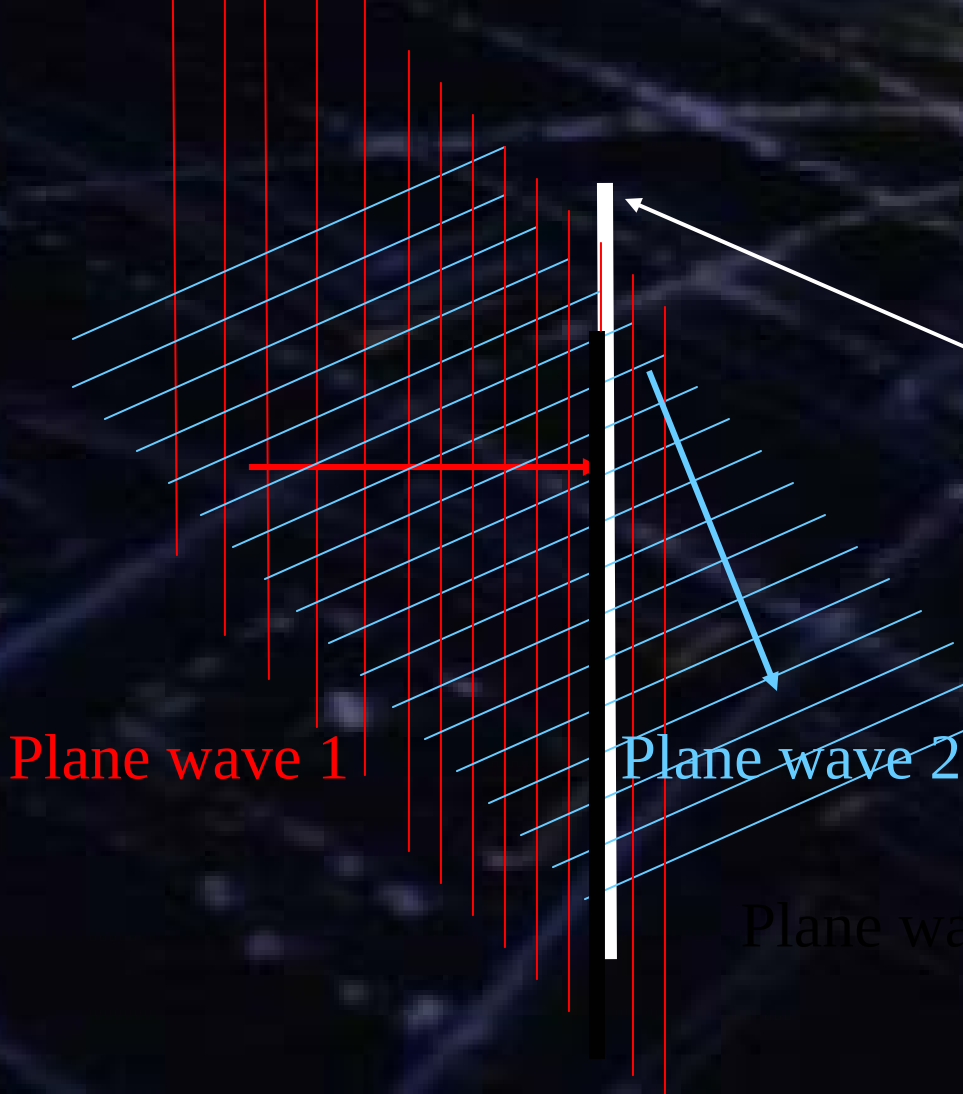
Cascading 3D Wedges to make 3D ‘Wedge-Cones’

$$y(n_1\Delta x_1, n_2\Delta x_2, n_3\Delta T_3) = x(n_1\Delta x_1, n_2\Delta x_2, n_3\Delta T_3) *** h_{F1}(n_1\Delta x_1, n_3\Delta T_3) *** h_{F2}(n_2\Delta x_2, n_3\Delta T_3)$$



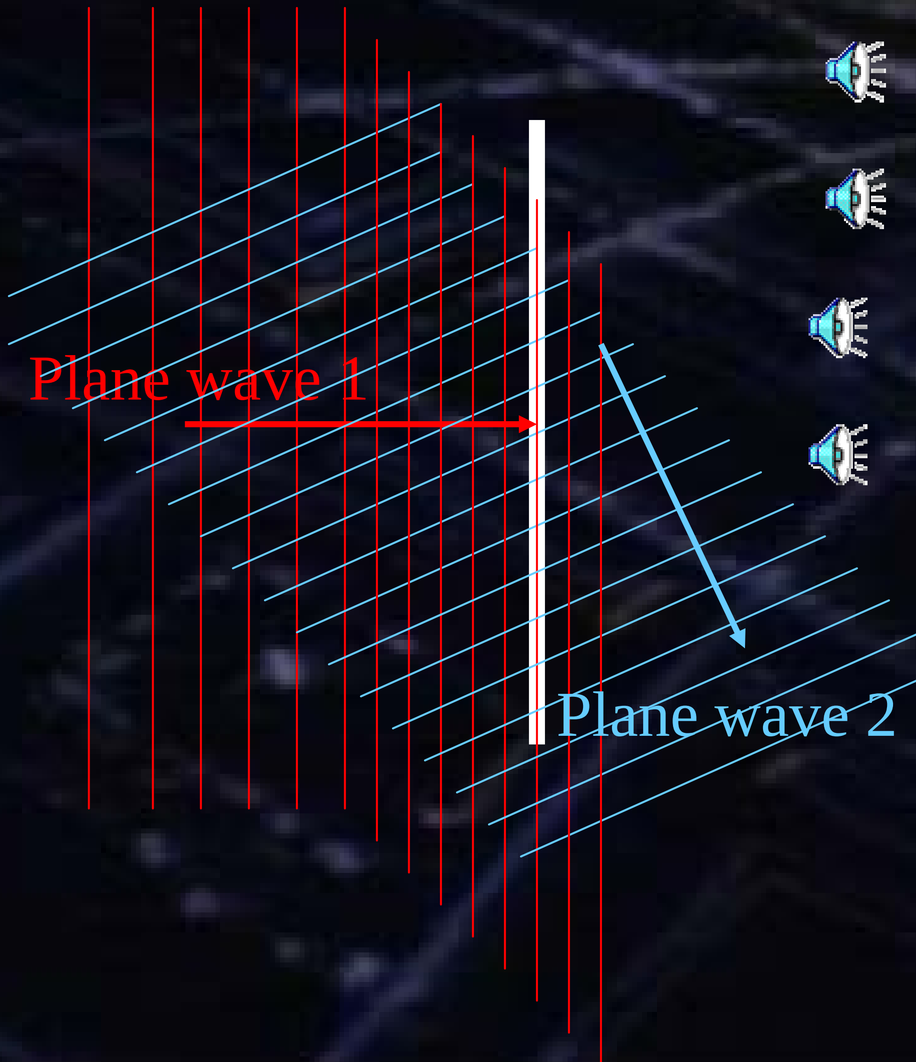


Application 4 : Acoustical Almost CD-Quality Spatio-temporal Jamming using 2D FIR Filter



- 89 sensor array
- Sensor spacing $\Delta x \approx 0.3$ cm
- Array length ≈ 30 cm
- Temporal sampling frequency $= \dfrac{1}{\Delta T} \approx 44$ KHz

Application 5: Acoustical 'Almost CD-Quality' Spatio-temporal Jamming using 2D FIR Filters - Simulated in Matlab



Wave 1 in Pass Band

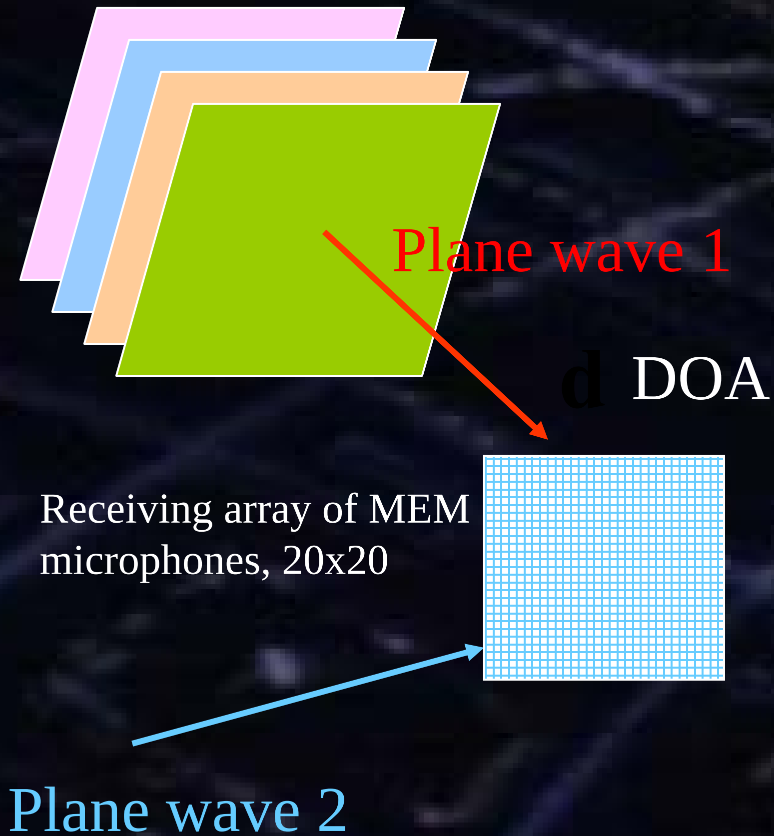
Wave 2 in Stop Band

INPUT – (Wave 1 + Wave 2)

OUTPUT $Y(N\Delta x, n_2\Delta T)$

- Uses compensation method
- Will allow some near field curvature – 5 degrees
- Stop band attenuation decreases for smaller separation of DOAs

Application 6: Acoustical 'Almost CD-Quality'Spatio-temporal Jamming using 3D FIR/IIR Filters IN PROGRESS



Wave 1 in Pass Band

Wave 2 in Stop Band

INPUT – (Wave 1 + Wave 2)

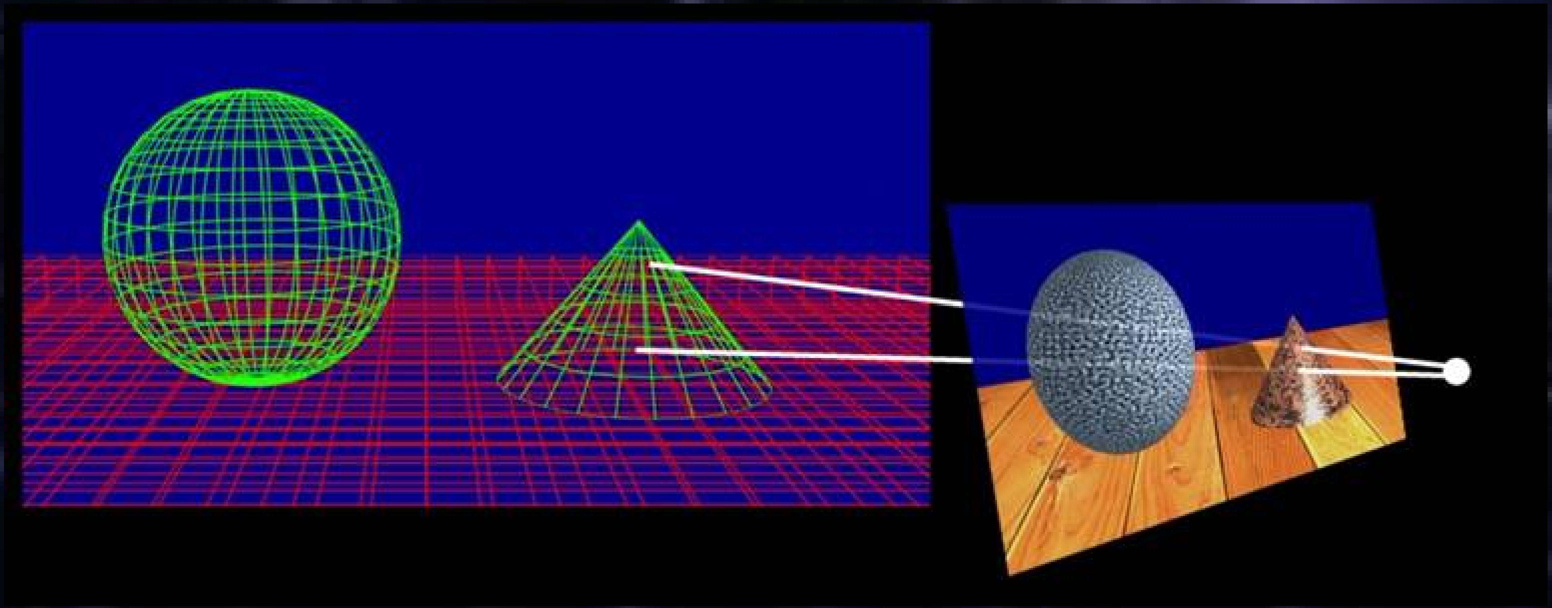
OUTPUT $Y(N_1\Delta x, N_2\Delta x, n_2\Delta T)$

- Uses compensation method
- Will allow some near field curvature – 5 degrees
- Stop band attenuation decreases for smaller separation of DOAs

Application 7: Object Extraction, Based on Depth, Using 4D IIR Filters Applied to 4D Light Fields

What is a 4D Light Field?

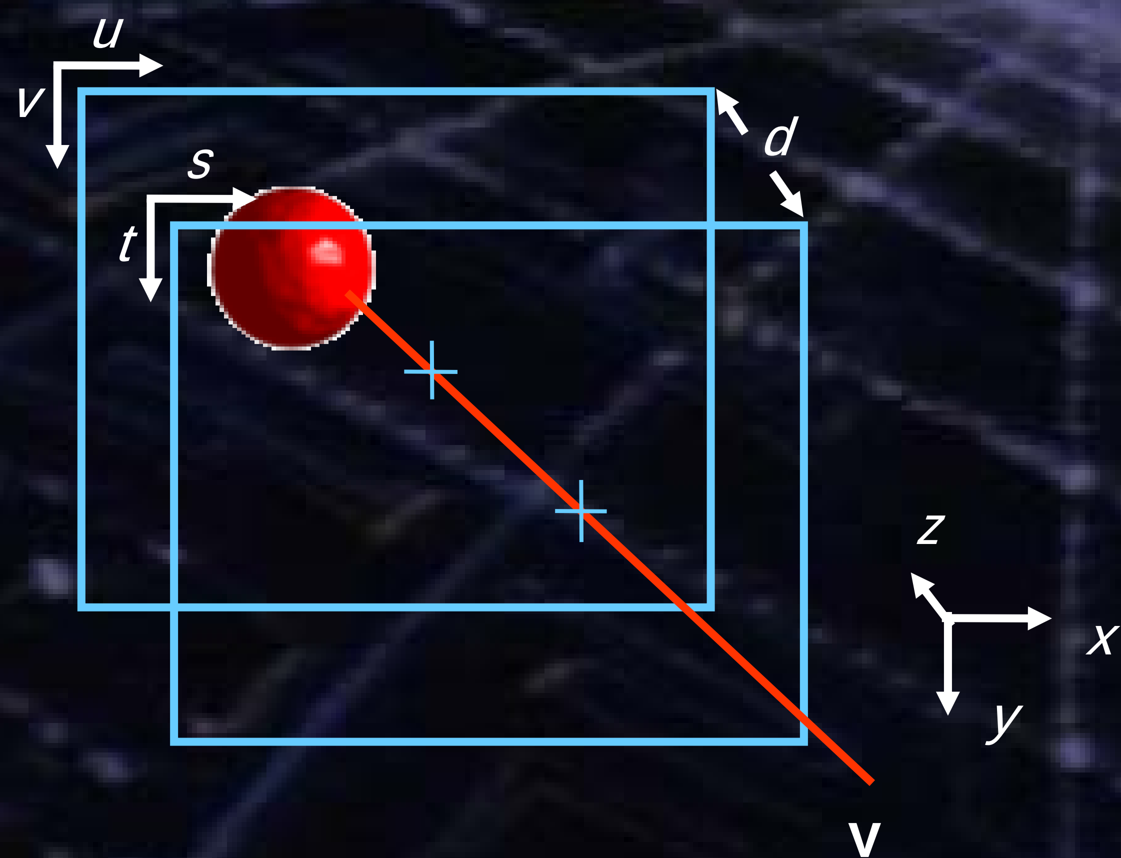
Ray tracers render images by modeling geometry (slow for complex scenes).



Light fields render images by modeling light rays (fast, independent of scene complexity, but large).

Parameterizing Light Rays

- Measure intersection of each ray with 2 reference planes.
- Ray value = $L(s, t, u, v)$, a 4D Signal
- Sample into a **4D** array.



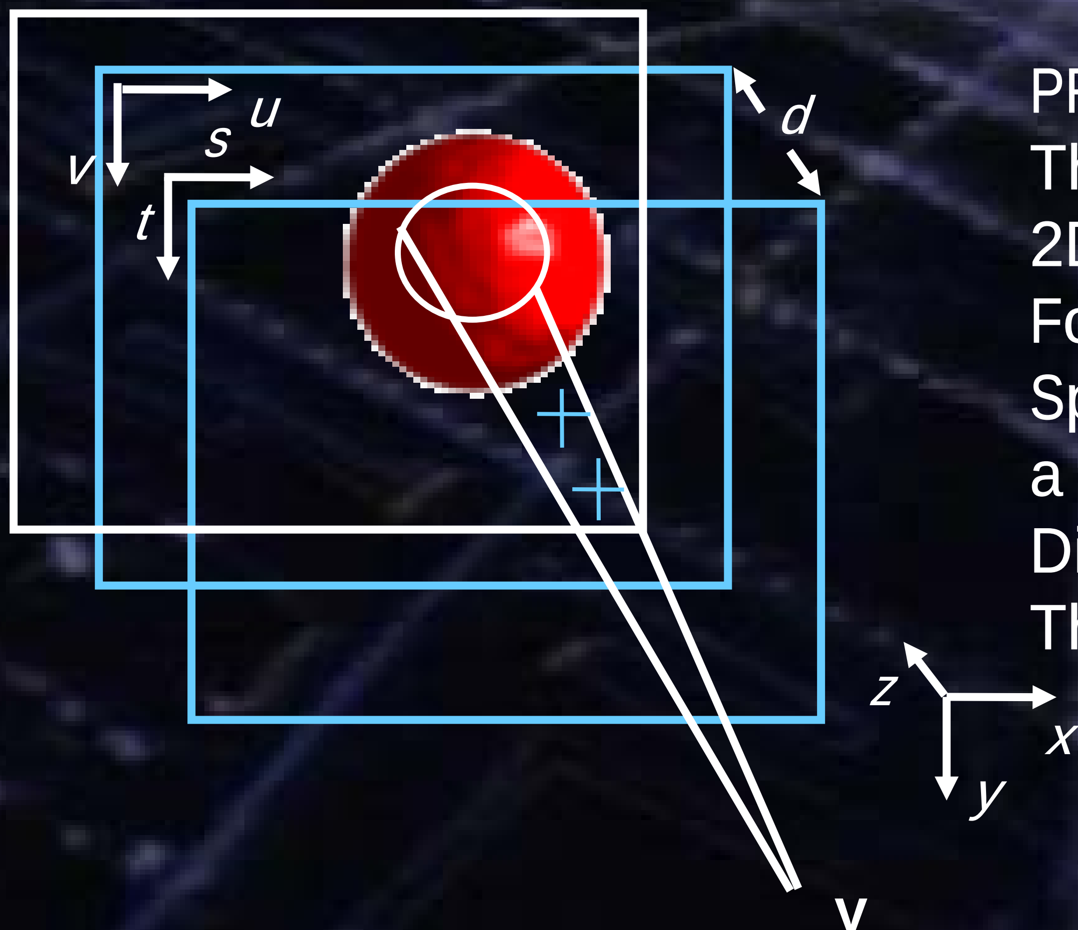
$$y(s,t,u,v) = x(s,t,u,v) ****h_{F1}(s,u) ****h_{F2}(t,v)$$

Output ray image

Input ray image

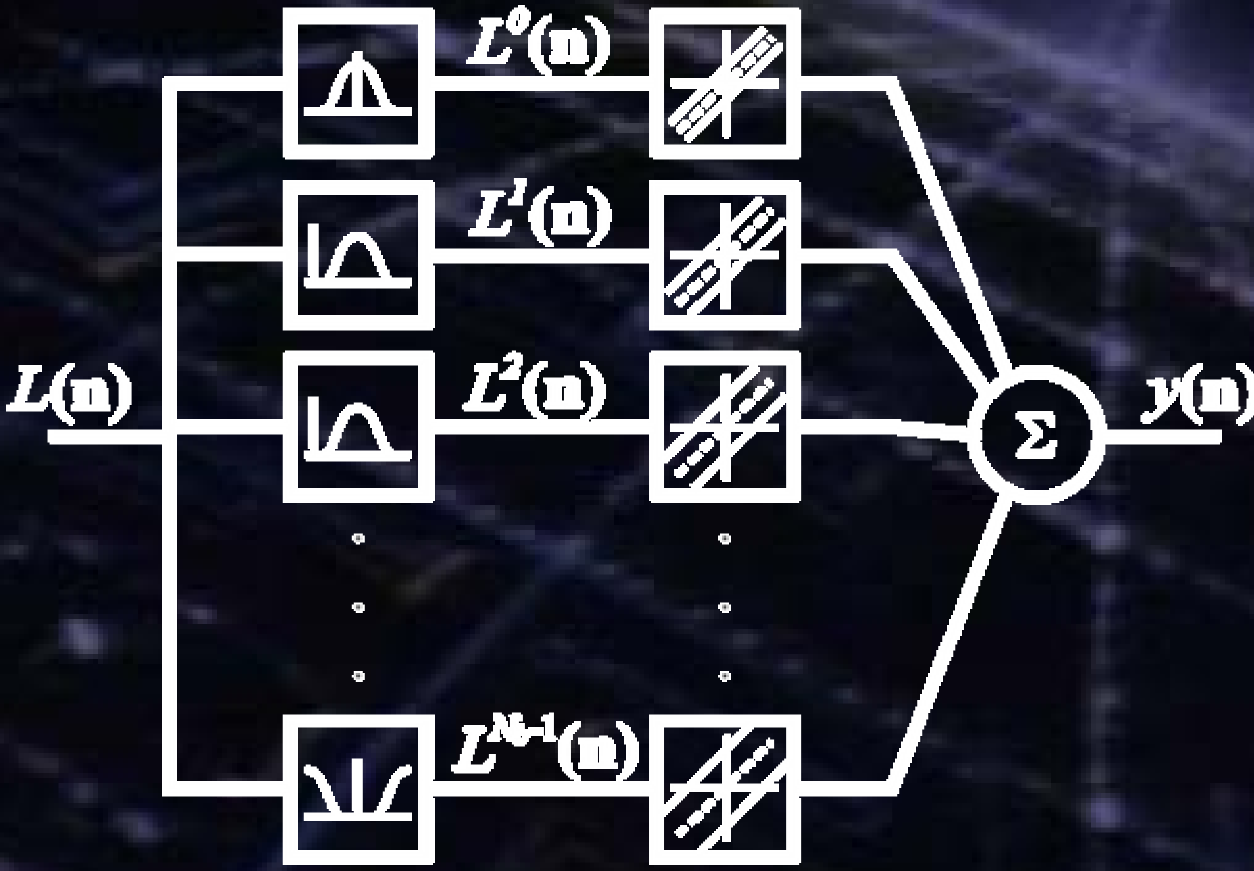
4D Wedge

4D Wedge



PRINCIPLE:
The Axes of the 2D Wedges Focus in a 3D Spatial Plane at a Fixed Distance From The Viewer

Structure of the 4D IIR Filter Bank Used to Make 4D Wedge Pass Bands

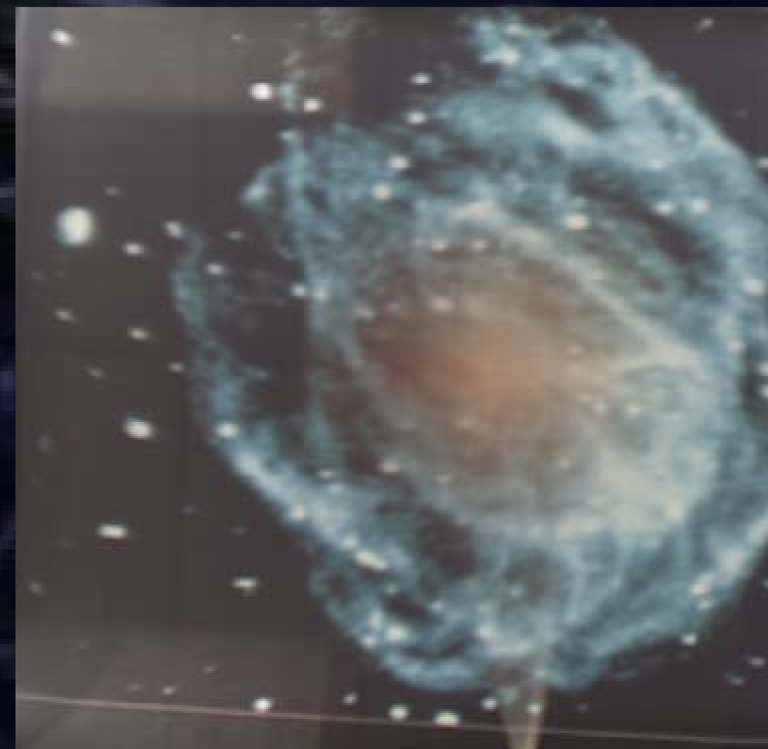


An example of light field measurement and rendering

Gantry image at $s,t=0$

Rendered image at novel camera position and orientation

Results of 4D IIR Filtering for Depth



4D IIR Broad Band Hyper Cone Filter Focused on Far Tree in 4D Lightfield with Time-varying Viewer Location

Input 4D Lightfield

Output 4D Lightfield

Real-time Hardware Implementations – Spatio-Acoustical Array Processing, Video Image Processing, Video Watermarking

- FPGA single-chip 2D and 3D FIR implementations up to 100 MHz (published) using scanned arrays
- TI DSP board implementations (ongoing) using Polyphase 3D Cone Filter Banks (published) for Object Tracking in Video (ongoing)
- Using Known Directionality of 2D/3D Impulse Response Transients to Reduce Complexity (2D published and 3D ongoing)

Ongoing Progress – Spatio-Acoustical Array Processing, Video Image Processing, Video Watermarking

- 4D Filters for Lightfield Processing with potential applications in computer vision
- 3D Watermarking Demonstration uses Directional 3D Filters to Recover Spatial-temporal Watermarks

Application： 3D Video Watermarking

Embedding Hidden Data
in Spatio-temporal
Sequences